PROBABILITY ANALYSIS OF THE MOTION OF PARTICLES IN A FLUIDIZED BED

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The velocity and acceleration of particles moving in a fluidized bed are analyzed by means of histograms and random functions.

One of the most important problems connected with the study of fluidized beds is the creation of a physical model, i. e., description of the bed by the equations of mathematical physics. The solution of the problem chiefly involves a knowledge of the nature of the motion of the gas in the bed itself and the kinematics and dynamics of its particles. The problem of the motion and interaction of the particles is a typical probality problem in relation to the primary impulse, the motion of the gas flow. So far, the ideas of probability theory have not been used to describe the behavior of the solid phase in a fluidized bed. The situation is complicated by the fact that attention has been concentrated on the averaged characteristics of the fluidized bed, where probability relations are inapplicable. Only when the local characteristics of the system are investigated is it possible to analyze the experimental data using the theory of probability and the theory of random functions. These local characteristics include the kinematic and dynamic parameters of motion of the bed particles.

Fluidization begins with displacement of the particles in a confined space [1], the actual motion of a particle being a random process. Analysis of the experimental data obtained by investigating this process using probability methods provides new information about the characteristics of the physical processes in the bed and the behavior of the solid phase during interaction with the gas flow. Naturally, probability relations can also be used to calculate the average kinematic characteristics of the particles.

The initial data were taken from a previous study in which the method of radioactively labeled particles was employed to investigate a monodisperse bed of spherical aluminosilicate catalyst [1–3].

The primary parameters were the components of the particle displacement in the vertical z and horizontal x, y directions, and time t. The kinematic parameters: vertical component of absolute particle velocity u_z and radial component u_R were calculated by numerical differentiation.

The experimental data were analyzed in two ways: by constructing histograms and by applying certain methods of the theory of random functions [4].

Figure 1 presents two histograms showing the density functions of the vertical component of the particle velocity in a monodisperse fluidized bed.

An analysis of the histograms for experiments with constant particle size and static bed height, but different gas velocities reveals the following points:

1. The density function of the vertical component of particle velocity is not normal. At a near-critical gas velocity this function is characterized by positive asymmetry. Its mode lies in the region of negative velocities; physically this means that the greatest probability of motion of the particles is downward.

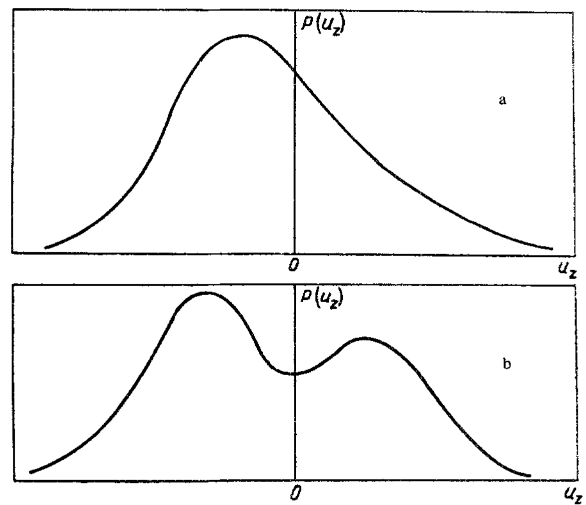

Fig. 1. Histograms of vertical component of absolute particle velocity u_z(mm/sec). Particle diameter 1.0–1.2 mm; gas velocity 0.456 m/sec (a) and 1.152 m/sec (b); critical gas velocity 0.310 m/sec; static height of bed 136 mm.

Hence we may conclude that the motion of the mass of particles as a whole obeys the law of conservation of momentum. From this it follows that a large number of particles moves slowly downward and a small number of particles rapidly upward.

2. With increase in gas velocity the density function of the vertical particle velocity component becomes bimodal, the ordinate of the "negative" mode being greater than that of the "positive" mode. Consequently, as before, motion downward is more probable than motion upward. The presence of bimodality indicates the appearance of a new random influence on the particle, that is, with increase in gas velocity the exchange of momentum between the particles due to collisions increases. Hence we may conclude that from a certain gas velocity the effect of collisions of the particles on their behavior is commensurable with the effect of the gas flow and the force of gravity. Only at gas velocities sufficiently close to critical is it possible to neglect the particle collision force.

3. With increase in gas velocity the "negative" mode is displaced to the left. In the region of positive

particle velocities (motion upward) the maximum of the density function degenerates.

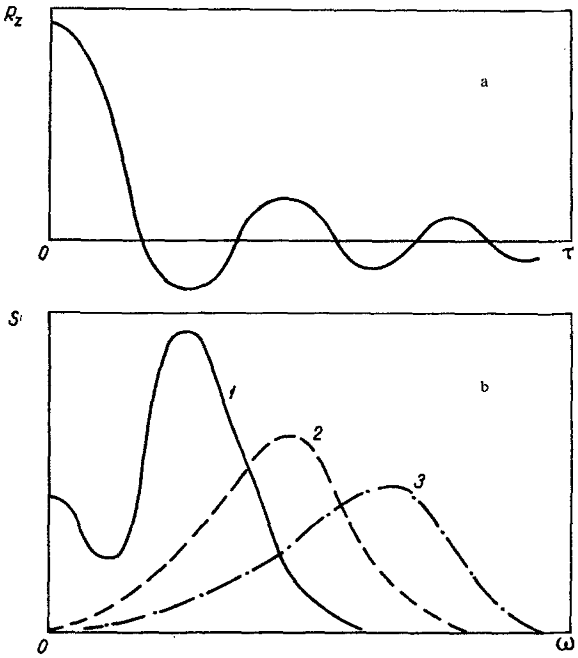

Fig. 2. (a) Correlation function $R_z(\tau)$ of the random function z(t) and (b) spectral density $S(\omega)$ of the random quantities z(t), $u_z(t)$, and $a_z(t)$. Particle diameter 1.0–1.2 mm; gas velocity 1.152 m/sec; critical velocity 0.310 m/sec; static height of bed 136 mm (R_z in mm^2; τ in sec; ω in Hz): 1) for $S_z(\omega)$; 2) $S_{u_z}(\omega)$; 3) $S_{a_z}(\omega)$.

4. The mathematical expectation of the investigated parameter, velocity u_z was found to be zero in all cases of analysis of the experimental data by the histogram method. Physically this means that the fluidized bed as a whole, regarded as a macrosystem consisting of moving particles, is stationary.

The dispersion of the distribution of the vertical particle velocity component increases with increase in gas velocity. This may be attributed to an increase in the rate of momentum transfer due to particle collisions.

The density function of the radial component of particle velocity is closely approximated by a normal law. Physically this means that radial motion is associated with the predominant influence on the particle of random factors, the probability of action of each of which is small.

Since the density function of the radial component is normal, whereas the density function of the vertical component differs sharply from a normal distribution (asymmetry, bimodality), the action of the gas flow on the particle, through a random factor, is oriented with respect to direction.

In analyzing one of the experiments we attempted to find the correlation of the vertical and radial components of particle velocity, which were treated as a system of two random quantities (u_z, u_R). In the first approximation it may be assumed that the correlation is relatively weak, and consequently these two random quantities may be regarded as independent. Then the density function of the two random quantities (u_z, u_R) will be equal to the product of the density functions of each of them [4].

In analyzing the experimental data by the methods of the theory of random functions we took as the initial random function z(t), the vertical coordinate vs. time.

Each experiment was conducted without variation of the external conditions in time. In the most general case the particle is acted upon by the following forces: interaction of particle with gas flow, collisions between neighboring particles, gravity, buoyant force. The first two are random quantities, and the probability of their acting on a particle does not change during the experiment. This makes it possible to treat the random function z(t) as stationary in the stochastic sense and ergodic, the latter property enabling the characteristics of the random function to be calculated from a single realization.

The correlation function and spectral density (Fig. 2) for one of the experiments were calculated on a high-speed computer.*

An analysis of the correlation function and spectral density of the random function z(t) reveals the following points:

1) The correlation function rapidly falls from its maximum value, equal to the variance of the random function z(t), to the first zero. Consequently, the motion of the particle is random, chaotic, and disordered.

2. The correlation function changes sign several times. Physically this means that there is an element of periodicity in the random motion of the particle. Thus, periodicity of motion of the particle has been proved by a strict mathematical method.

3. While changing sign, the correlation function tends to zero. This indicates the ergodicity of the random function.

The spectral density of the random function z(t) has two maxima: the first at a frequency $\omega = 0$ and the second at $\omega = \omega_1$. A physical explanation of the significance of these maxima is given below.

Analysis of the experimental data using the theory of random functions makes it possible to obtain spectral densities for the vertical components of the velocity and acceleration, which are also treated as random functions.

In order to obtain the velocity and acceleration knowing the time dependence of the coordinate, we apply the differentiation operator. We assume that this operator corresponds to a stationary linear system whose input is z(t) and whose output is $u_z(t)$. Then the amplitude-frequency characteristic of this stationary linear system is given by

$$\Phi(i\omega) = \omega.$$

*E. M. Ruzhnikov was responsible for the programming and computations.

Consequently, the spectral density at the output of the linear system

$$S_{u_z(t)}(\omega) = |\Phi(i\omega)|^2 S_{z(t)}(\omega) = \omega^2 S_{z(t)}(\omega).$$

Again applying the differentiation operator, we obtain

$$S_{a_z(t)}(\omega) = \omega^2 S_{u_z(t)}(\omega) = \omega^4 S_{z(t)}(\omega).$$

Thus, obtaining the spectral density of $u_z(t)$ and $a_z(t)$ reduces to multiplying the spectral density of z(t) by ω^2 and ω^4, respectively.

The graphs of the spectral densities of the random functions $u_z(t)$, $a_z(t)$ have a single maximum (Fig. 2b).

As a result of the spectral densities obtained, each random function z(t), $u_z(t)$, and $a_z(t)$ can be represented in canonical form [4], i.e., in the form of a Fourier series,

$$\sum_{k=0}^{\infty} (U_k \cos \omega_k t + V_k \sin \omega_k t),$$

where the series coefficients U_k and V_k are ordinary random quantities which for the same term of the series k have the same variance, determined by the spectral density curve and the mathematical expectation, namely, zero. Mathematically this means that there is a possibility of expressing the most probable coordinate, velocity and acceleration as a function of time.

Consequently, from an analysis of the spectral density curves of the random functions z(t), $u_z(t)$, and $a_z(t)$, we can draw the following conclusions (Fig. 2b):

1. Since at $\omega = 0$ the spectral density of the random function z(t) has a maximum, this random function has as a component the ordinary random quantity

$$z(t) = U_0 + \sum_{k=1}^{\infty} (U_k \cos \omega_k t + V_k \sin \omega_k t).$$

2. The presence of a second maximum of the spectral density curve of the random function z(t) at $\omega = \omega_1$ indicates the presence of a carrier (fundamental) frequency of particle oscillation from the distributor grid to the free surface of the bed.

3. The existence of maxima on the spectral density curves of the random functions $u_z(t)$ and $a_z(t)$ at $\omega = \omega_2$ and $\omega = \omega_3$ indicates the presence of fundamental frequencies of variation of particle velocity and acceleration.

Thus, the use of stochastic (probability) concepts and the theory of random functions appears to be very promising in relation to fluidization processes. With their help it is possible not only to reduce the experimental data for purposes of generalization, but also to obtain new information on the physical phenomena in fluidized beds. Some of this information cannot be obtained directly from experiment (in our case the quantitative effects of particle collisions).

The rationality of the method employed is confirmed by its successful application in areas related to fluidization [5, 9].

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